**Problem sheet 1: Magnetization, diamagnets and paramagnets** **23.10.2024**

You can use both the SI or CGS units, but you may find CGS easier whenever magnetic moments and magnetic susceptibilities are involved.

Remember that in 1.1 and 1.2 one can safely assume $B \simeq H$ in CGS (or $B \simeq \mu_0 H$ in SI), because sample magnetization is very small compared to the field strength.

### 1.1. Diamagnetic levitation (7 P)

Consider Geim's experiment on diamagnetic levitation:

(a) Derive the levitation condition. The force acting on the sample due to its interaction with the magnetic field is given by $\mathbf{F} = V(\mathbf{M}\boldsymbol{\nabla})\mathbf{B}$. Assume $\mathbf{B} \| z$ (vertical).

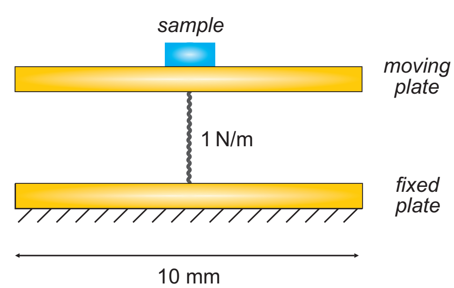
(b) Estimate the field gradient required for levitating a piece of chalk ($CaCO_3$) in a magnet with the constant field of 14 T.

(c) Compare with the field gradient required for levitating a frog.

Determine the diamagnetic susceptibilities using Pascal's constants. Assume that chalk is $CaCO_3$ with the 100 % density (real chalk is somewhat less dense), whereas frog is pure water ($H_2O$). You can find the Pascal's constants in J. Chem. Education 85, 523 (2008) (their units of emu/mol are cm$^3$/mol, CGS).

### 1.2. Application of a Faraday magnetometer (8 P)

A Faraday magnetometer consists of two square-shaped plates with the lateral dimensions of 10 mm. While the bottom plate is fixed, the top plate is attached to a spring with the spring constant of 1 N/m. A 10 mg sample of $CuSO_4 \cdot 5H_2O$ is placed onto the top plate. Then the magnetic field of 10 kOe is switched on, augmented by the field gradient of $dH/dz = 300$ Oe/cm, and the capacitance is measured at several temperatures (in picoFarad):




10 K: 0.4428098 pF, 20 K: 0.4427599 pF, 30 K: 0.4427433 pF

The measurement without the magnetic field returns the capacitance of 0.4427100 pF.

(a) From the force balance between $F = \mu_m \times dH/dz$ (force due to the magnetic field) and the elastic force of the spring, determine the magnetic moment of the sample at each temperature.

(b) Calculate the molar magnetic susceptibility ($\chi_m$) at each temperature

(c) Determine the Curie constant and the effective magnetic moment of $Cu^{2+}$ (in units of $\mu_B$).

Assume that the measurement is performed with a low pressure of helium gas, such that $\varepsilon \simeq 1$. Neglect the contribution of the sample to the capacitance. For simplicity, we also assume that the spring is located in the center of the plates, and the sample is perfectly aligned to the center too, such that the torque is zero.

### 1.3. Algebra of the Brillouin function (5 P)

Consider the Brillouin function:

$$B_J(x) = \frac{2J+1}{2J} \times \operatorname{cth} \frac{(2J+1)x}{2J} - \frac{1}{2J} \times \operatorname{cth} \frac{x}{2J}$$

that enters the magnetization of a paramagnet,

$$M = \frac{N}{V} g\mu_B J \times B_J(x)$$

where $x = g\mu_B JB/(k_B T)$.

(a) explore the classical limit, $J \to \infty$, and show that $B_J(x)$ becomes similar to the Langevin function

(b) explore the high-field limit, $x \to \infty$, and show that $B_J(x)$ saturates to a finite value; what is the saturation magnetization in this case?

(c) explore the low-field limit, $x \ll 1$, and show that the Curie law is recovered. To this end, use $\mathrm{cth}\, x \simeq 1/x + x/3$ (this representation can be obtained from the Taylor expansion of $\mathrm{th}\, x$).