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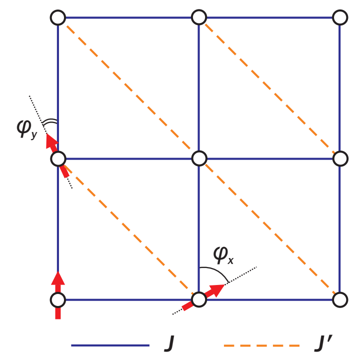
Problem sheet 5: Magnetic order and magnetic excitations **12.01.2025**

You can use both the SI or CGS units, but you may find CGS easier whenever magnetic moments and magnetic susceptibilities are involved.

5.1. Helical order (8 P)



Consider an anisotropic triangular antiferromagnet that can be represented by a square lattice with one diagonal coupling (see the figure). Assume a generic magnetic structure where the adjacent spins along x and y form the angles φ_x and φ_y, respectively.

(a) Write energy using the Heisenberg model, and minimize it with respect to φ_x and φ_y. In which range of J'/J would you expect the helical order?

(b) What kind of magnetic order would you expect in the ideal triangular antiferromagnet, $J = J'$?

(c) What is the propagation vector at $J = J'$?

5.2. Spin waves in antiferromagnets (8 P)

Derive the spin-wave dispersion in a two-sublattice antiferromagnet. To this end, consider a chain of antiferromagnetically coupled spins (S) and use the condition $S_p^z = (-1)^p S$.

(a) Using the general equation of motion derived in the lecture,

$$\hbar \frac{d\mathbf{S}_p}{dt} = J(\mathbf{S}_p \times \mathbf{S}_{p-1} + \mathbf{S}_p \times \mathbf{S}_{p+1})$$

write the equations of motion for S_p^x and S_p^y, as well as S_{p+1}^x and S_{p+1}^y.

(b) Transform these four equations into the two equations of motion for S_p^+ and S_{p+1}^+ where $S^+ = S^x + iS^y$.

(c) Use the ansatz

$$S_p^+ = u\, e^{i(pqa-\omega t)}, \quad S_{p+1}^+ = v\, e^{i[(p+1)qa-\omega t]}$$

to obtain the spin-wave dispersion relation for an antiferromagnet,

$$\hbar\omega = 2JS \sin qa$$

5.3. Domain walls (4 P)

(a) Use spin-wave stiffness of the metallic iron, $A = 280\,\text{meV}/\text{Å}^2$, to determine the effective exchange coupling J. Assume the spin-wave dispersion for a chain, $\hbar\omega = A\,q^2$ with $A = JS\,a^2$ as shown in the lecture, the lattice constant $a = 2.86\,\text{Å}$, and $S = 1$.

(b) The old measurements from 1950's return the anisotropy constant of $K = 5 \times 10^5\,\text{erg/cm}^3$. Bring it to the more modern units and estimate the domain wall thickness, as well as the domain wall energy of the metallic iron.