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Problem sheet 1: Meissner effect and thermodynamics **7.04.2024**

Solving the problems requires the following properties of superconductors:

	T_c [K]	$H_c(0)$ [Oe]	ρ (g/cm^3)	M (g/mol)
Sn	3.722	305	7.27	118.7
Pb	7.196	803	10.66	207.2

Here, T_c is the critical temperature at zero field, $H_c(0)$ is the critical field at $T = 0$, ρ is density, and M is molar mass. Critical fields at intermediate temperatures can be calculated using the empirical formula

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

Give your solution in the SI units, unless requested otherwise.

1.1. Superconductor vs. applied field (6 P)

Consider a thin lead disc characterized by the demagnetization factors $f \simeq 0$ and $f = 0.9$ when the field is applied parallel and perpendicular to the surface, respectively.

(a) Sketch magnetic field lines and calculate supercurrent (its linear density, as shown in lecture 1) for the magnetic field of 100 Oe applied *parallel* to the disk surface at $T = 4.2$ K.

(b) Stay at $T = 4.2$ K and keep boiling helium away, but now apply the field *perpendicular* to the disk surface. Sketch magnetic field lines for three situations: $H = 50$ Oe, $H = 300$ Oe, and $H = 700$ Oe.

(c) Sketch the magnetization curves (M vs. H) for both field orientations. Give characteristic values of the magnetization (in emu/cm^3) and field (in Oe).

1.2. Magnetic levitation: how high can it go? (6 P)

Consider a dense sphere of lead with the radius of 1 mm.

(a) Calculate magnetic moment of this sphere in the applied field of 200 Oe. Express the value in both SI and CGS units.

(b) Use the interaction energy $\mathcal{E} = -V\,\mathbf{M}\cdot\mathbf{B}$ to obtain an expression for the lifting force, $\mathbf{F} = -\mathrm{grad}\,\mathcal{E}$, assuming that magnetization is constant within the sphere. Derive the condition for magnetic levitation.

(c) Place the sphere above the magnet. What is the equilibrium distance between the sphere and the magnet surface if magnetic field decreases as d/z, where z is the distance to the surface and $d = 1000$ Oe cm?

1.3. Work to do and heat to release (8 P)

(a) Calculate the maximum condensation energy ($\Delta G = G_n - G_s$) of tin in zero magnetic field. Express the result in eV/atom and compare it with the typical atomic energies.

(b) Take the derivative of ΔS and obtain an expression for the specific heat jump, ΔC_p. Calculate ΔC_p for tin in zero field, express the result in mJ/mol K.

(c) Specific heat of the normal state is well approximated by $C_n(T) = \gamma T$ where $\gamma = 1.78$ mJ/mol K^2 is the Sommerfeld coefficient for tin. In contrast, specific heat of superconducting tin is roughly exponential, $C_s(T) = A\exp(-\Delta/T)$ with $A = 56.3$ mJ/mol K. Determine Δ.

(d) How much heat does 1 mol of tin release upon entering the superconducting state when it is cooled down in the applied field of 200 Oe?

(e) Obtain an expression for the temperature dependence of the latent heat, $Q(T)$. Sketch $Q(T)$ over the whole range of interest, between 0 and T_c. Compare to the experimental result that was shown in lecture 2.