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Problem sheet 3: Type-II superconductors **16.05.2024**

Solving the problems requires the following properties of superconductors:

	T_c [K]	$H_c(0)$ [Oe]	ρ (g/cm^3)	M (g/mol)
Pb	7.196	803	10.66	207.2

Here, T_c is the critical temperature at zero field, $H_c(0)$ is the critical field at $T = 0$, ρ is density, and M is molar mass. Critical fields at intermediate temperatures can be calculated using the empirical formula

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

Give your solution in the SI units, unless requested otherwise.

3.1. Critical fields and vortex parameters (8 P)

At zero temperature, Nb_3Sn has the critical fields $B_{c1} = 26\,\mathrm{mT}$ and $B_{c2} = 29\,\mathrm{T}$.

(a) Determine the coherence length (ξ), penetration depth (λ), Ginzburg-Landau parameter (κ), and thermodynamic critical field (B_c).

(b) Determine the vortex energy and compare it to the condensation energy assuming that the core of the vortex is in the normal state. Define the core as the cylinder with the radius of ξ.

Note that two solutions for κ are possible mathematically, but only the one with $\kappa \gg 1$ is relevant to Nb_3Sn (see also lecture 6).

3.2. Superconducting magnet (6 P)

Consider a coil of superconducting material with the length of 36 cm, 12000 windings, and the diameter of 10 cm.

(a) Calculate the inductance of this coil

(b) A voltage of 1.5 V is applied to charge the coil. How long will it take to reach the current of 150 A?

(c) What magnetic field will be generated by this current?

(d) Assume that the superconducting wire experiences the same magnetic field as in the center of the solenoid. Determine lattice parameter of the vortex lattice.

Hint: recall the expressions for the inductance and magnetic field of a solenoid from your experimental physics lectures.

3.3. Surface energy (6 P)

Calculate the surface energy of the interface between the superconducting and normal states of lead at 2 K. Assume $\xi \gg \lambda$ and use the order parameter $\psi(x) = \tanh\left(x/\sqrt{2}\,\xi\right)$ from lecture 5 as well as the coherence length $\xi(2\,\mathrm{K}) = 106\,\mathrm{nm}$ (see problem 2.2).

Compare the result to the surface tension of water at room temperature.

Hint: The integral can be calculated in WolframAlpha or found in Wikipedia.