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Problem sheet 2: Integer quantum Hall effect **28.10.2024**

2.1. Cyclotron motion

Consider a classical particle of mass m and charge $q = -e$ moving in two dimensions in the presence of a perpendicular magnetic field of magnitude $\mathbf{B} = B\,\mathbf{e}_z$, so that this particle executes the standard 2D cyclotron motion. Answer the following questions about the cyclotron motion:

(a) Consider the two cases, 1 and 2, in which the particle is prepared with two different initial velocities, v_1 and v_2, that satisfy $v_2 = 2\,v_1$. What is the proportion of the periods to complete the cyclotron motion between these two cases? What is the proportion of the radii of the cyclotron orbits between the two cases?

(b) Find a general relation between the classical kinetic energy of the particle and the classical radius of its cyclotron orbit.

(c) Assuming that energy levels are quantized according to $E_n = \hbar\omega_c(n + 1/2)$ where ω_c is the cyclotron frequency, find a heuristic semiclassical relation for the cyclotron radius, R_n, of the n-th orbital, using the result from part (b).

2.2. Electron velocity and conductivity

Consider a classical particle of mass m and charge q moving in two dimensions in the presence of a perpendicular magnetic field, $\mathbf{B} = B\,\mathbf{e}_z$, and a constant in-plane electric field, $\mathbf{E} = E_x\mathbf{e}_x$.

(a) Write and solve the classical equations of motion for the particle in this case.

(b) Compute the time average of the velocity over one cyclotron period, namely:

$$\langle v_x \rangle = \frac{1}{T}\int_t^{t+T} v_x(t)\,dt,$$

$$\langle v_y \rangle = \frac{1}{T}\int_t^{t+T} v_y(t)\,dt.$$

(c) Define the electrical current density as $\boldsymbol{j} = -en\langle v \rangle$ (here, we choose $q = -e$ for an electron) and compute the classical conductivity matrix, namely, the linear coefficients relating j_x and j_y to E_x.

2.3. Equation of motion

Consider a quantum particle of mass m and charge $q = -e$ moving in two dimensions in the presence of a perpendicular magnetic field, $\mathbf{B} = B\,\mathbf{e}_z$.

(a) The quantum velocity operators can be defined as follows,

$$\hat{v}_x = \frac{\hat{p}_x + eA_x(\hat{x},\hat{y})}{m}, \qquad \hat{v}_y = \frac{\hat{p}_y + eA_y(\hat{x},\hat{y})}{m}$$

where $\hat{p}_{x,y}$ are the standard momenta that obey the canonical commutation relations with the position operators $\hat{x}$ and $\hat{y}$. Compute the commutator of these velocity operators, $[\hat{v}_x, \hat{v}_y]$.

(b) Derive the Heisenberg equations of motion for the above velocities, for a Hamiltonian that includes the electric field along x,

$$\mathcal{H} = \frac{m}{2}(\hat{v}_x^2 + \hat{v}_y^2) + eE_x\hat{x}.$$

Submit the written solution to Problem 2.3 in the class on 6.11.24 at 15:15
Tutorial session on 6.11.24 at 15:15

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The Heisenberg equation of motion is given by

$$\frac{d\hat{O}}{dt} = \frac{i}{\hbar}[\mathcal{H}, \hat{O}]$$

where $\hat{O}$ is an arbitrary operator.

(c) Compute the equations of motion for the expectation values of the $\hat{v}_x$ and $\hat{v}_y$ operators in an arbitrary quantum state. Compare these equations of motion to the classical ones from the problem 2.2. What conductivity would you expect in the quantum case, and how is it related to the classical one?