Advanced Solid State Physics
WS 24/25, Uni Leipzig

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Problem sheet 5: Topological insulators, Weyl fermions **27.11.2024**

5.1. Time-reversal symmetry

(a) Show that in the case of spin-$\frac{1}{2}$ the time-reversal symmetry operator can be represented as

$$\hat{T} = -i\sigma_y \hat{K}$$

where σ_y is the Pauli matrix, and $\hat{K}$ is the complex conjugation. Use $\hat{T} = e^{i\pi \hat{S}_y}\hat{K}$ as given in the lecture, and try Taylor expansion.

(b) The low-energy Hamiltonian of the Haldane and Kane-Mele models is given by

$$\begin{aligned} \mathcal{H}(K + \mathbf{p}) &= \quad 3\sqrt{3}\, t_2 \sin\varphi - v(\hat{p}_x\sigma_x + \hat{p}_y\sigma_y) \\ \mathcal{H}(K' + \mathbf{p}) &= -\, 3\sqrt{3}\, t_2 \sin\varphi - v(\hat{p}_x\sigma_x - \hat{p}_y\sigma_y) \end{aligned}$$

in the vicinity of K and K', respectively. The time-reversal operation transforms K into K'. Show explicitly that the Haldane model is not time-reversal invariant, whereas time-reversal invariance is restored in the Kane-Mele model.

5.2. Lattice model of Weyl semimetal

Consider the Bloch Hamiltonian

$$h(\mathbf{k}) = t \sin k_x\, \sigma_x + t \sin k_y\, \sigma_y + t(2 + \gamma - \cos k_x - \cos k_y - \cos k_z)\, \sigma_z$$

(a) Find the dispersion relation of the bands. Assuming $|\gamma| < 1$, what are the points in the first Brillouin zone at which the bands touch each other? Call these points $\mathbf{k}_W$.

(b) Expand the Bloch Hamiltonian around the $\mathbf{k}_W$ points to leading order in momentum difference to get the effective long wavelength Hamiltonian. (This should resemble the Hamiltonian for Weyl fermions discussed in the class.)

(c) Consider a generic perturbation to the 2×2 Bloch Hamiltonian $h(\mathbf{k})$ given by

$$\delta h(\mathbf{k}) = b_0(\mathbf{k})\mathbb{1} + b_x(\mathbf{k})\, \sigma_x + b_y(\mathbf{k})\, \sigma_y + b_z(\mathbf{k})\, \sigma_z \tag{1}$$

What is the effect of this perturbation on the effective Hamiltonian from part (b)? Specifically, does the effective Hamiltonian of $h(\mathbf{k}) + \delta h(\mathbf{k})$ still remain Weyl-like?

Hint: Taylor expand the b functions

5.3. Discrete symmetries and Weyl nodes

We saw in the lectures that at least one of the time-reversal symmetry (TRS) and inversion symmetry (IS) needs to be broken to generically ensure band touchings in 3D. Find the minimum number of Weyl nodes that a 3D system can host in the following scenarios:

(a) TRS is intact but IS is broken

(b) TRS is broken but IS is intact

Hint: Consider the effect of these symmetries on the band structure

Submit the written solution to Problem 5.2 in the class on 4.12.24 at 15:15
Tutorial session on 4.12.24 at 15:15