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Advanced Solid State Physics
WS 24/25, Uni Leipzig

Problem sheet 6: Topological semi-metals **6.12.2024**

6.1. Getting the Weyl nodes at the Fermi level

(a) Show that the contribution of states near a 3D Weyl point at energy E_0 to the density of states $N(E)$ is quadratic in $|E - E_0|$. The density of states is given by

$$N(E) = \frac{1}{(2\pi)^3} \int \delta(E - E_{\mathbf{k}})\, d\mathbf{k}$$

where $E_{\mathbf{k}}$ is the band energy.

(b) Ferromagnetic $K_2Mn_3(AsO_4)_3$ is expected to feature the Weyl nodes along $\Gamma - Z$ located 30 meV below the Fermi level (see Phys. Rev. Lett. 128, 176401 (2022)). Estimate how many holes (per formula unit) should be doped into $K_2Mn_3(AsO_4)_3$ in order to tune the Fermi level to the position of these Weyl nodes. Remember that there are two equivalent Weyl nodes at this energy. Neglect the contributions from other bands.

6.2. Hall conductivity of a Weyl material

Consider a minimal continuum model of a Weyl semi-metal (WSM) in 3D with a pair of Weyl nodes at $(0, 0, \pm k_0)$,

$$h(\mathbf{k}) = \begin{pmatrix} v_F\, \mathbf{k} \cdot \boldsymbol{\sigma} - v_F k_0 \sigma_z & 0 \\ 0 & -v_F\, \mathbf{k} \cdot \boldsymbol{\sigma} - v_F k_0 \sigma_z \end{pmatrix} \tag{1}$$

We will work out the Hall conductance σ_{xy} of this model. Hall conductance of a 2D material is related to the Chern number by

$$\sigma_{xy}^{\mathrm{2D}} = C \frac{e^2}{h} \tag{2}$$

Our model of the WSM can be imagined as a stack of independent 2D Chern insulators (CI) along k_z, i.e., at each k_z the Hamiltonian is a 2D CI with a certain Chern number. Each one of these 2D insulators contributes to the Hall conductivity in the (x, y) plane. The total Hall conductivity of the WSM is then the integral of the 2D Hall conductivity for all k_z. Show that this total conductivity is

$$\sigma_{xy}^{\mathrm{3D}} = \frac{2k_0}{2\pi} \frac{e^2}{h} \tag{3}$$

This is a simple example of a more general result: the Hall conductance of a WSM is proportional to the momentum difference between the Weyl nodes.

6.3. Particle Hole symmetry of pairing Hamiltonians

A single-particle Bloch Hamiltonian $\mathcal{H}_{\mathbf{k}}$ is said to have *particle-hole* symmetry if for some unitary matrix $\mathcal{U}$

$$\mathcal{U}^\dagger\, \mathcal{H}_{\mathbf{k}}^* \mathcal{U} = -\mathcal{H}_{-\mathbf{k}} \tag{4}$$

(a) For such a system show that if a state exists at energy E there must be another state at energy $-E$.

(b) For a two-band Bloch Hamiltonian parametrized as

$$\mathcal{H}_{\mathbf{k}} = \varepsilon_{\mathbf{k}} \hat{1} + d_x(\mathbf{k})\sigma_x + d_y(\mathbf{k})\sigma_y + d_z(\mathbf{k})\sigma_z$$

what properties should ε and $\vec{d}$ satisfy in order to have particle-hole symmetry? Consider $\mathcal{U} = \sigma_x$

(c) We saw in the class that the Kitaev chain Hamiltonian corresponds to $\varepsilon = 0$ and

$$\vec{d}(k) = (0,\, 2\Delta \sin k,\, -2t \cos k - \mu).$$

Does it have particle-hole symmetry?

Submit the written solution to Problem 6.2 in the class on 18.12.24 at 15:15
Tutorial session on 18.12.24 at 15:15